The Economic Club

The Theory of Interstellar Trade

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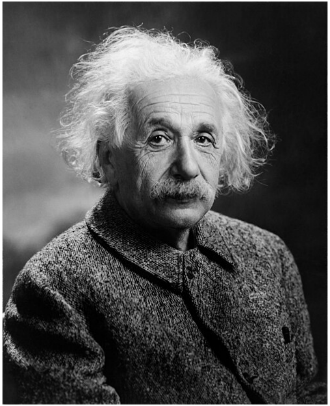
What if we trade with other planetary systems?

- In international trade, trade costs depend on how long it takes goods to be transported from one country to another.
- Interstellar trade is different. Distances are orders of magnitude larger (light years), so ships must travel at significant fractions of the speed of light.
- At those speeds, Albert Einstein's special relativity starts mattering: the passage of time during transit is no longer obvious.

> "It should be noted that, while the subject of this article is silly, the analysis actually does make sense. This article, then, is a serious analysis of a ridiculous subject, which is of course the opposite of what is usual in economics."

Recapping Special Relativity

- The speed of light in a vacuum, c ($\approx$ 300.000 km/s), is the same for every observer, no matter how fast they are moving.
- Ordinary velocities add up: a ball thrown forward on a moving train goes faster to a platform observer. For light, it's always c.
- To keep c constant across observers moving at different speeds, something else has to give: space and time.



Time Dilation

FIGURE 2
Rotation of the Imaginary Time Axis in Minkowski Space-Time

- Two consequences:
  1. a clock in motion relative to you ticks slower than an identical clock at rest next to you (*time dilation*).
  2. observers moving relative to each other can disagree about which of two distant events happened first (*relativity of simultaneity*).

The setup

The notation used will be the following:

p_E, p_T	Price of Earth and Trantorian goods on Earth
p_E^*, p_T^*	Price of Earth and Trantorian goods on Trantor
r, r^*	Interest rates on Earth and Trantor

N will be the time required to travel between Earth and Tranton for an observer in the Earth-Trantor intertial frame. Finally,

$$\bar{n} = N \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

Is the time required to travel between Earth and Trantor for an observer aboard a spaceship travelling at speed v.

An interstellar transaction

- For now, let $r = r^*$.
- Suppose an entrepreneur on Trantor is considering the following project:
  1. Making an initial expenditure $c + q_T^* \cdot p_T^*$ to ship q_T^* goods to Earth.
  2. The Trantorian goods are exchanged for Earth goods, where $q_E = \frac{q_T^* \cdot p_T}{p_E}$ and shipped back to Trantor.
  3. Upon arrival at Trantor, they are sold at price p_E^* for a total revenue of $\frac{p_E^* q_T^* p_T}{p_E}$.
- How does the entrepreneur evaluate whether this is a profitable project?
- Recall the net present value (NPV) formula, where a project is profitable if:

$$\Pi \geq C \cdot (1 + r)^T$$

The fundamental problem: Space-time

- One possibility is to take the NPV in the Earth-Trantor intertial frame:

$$\frac{p_E^* q_T^* p_T}{p_E} \geq (c + q_T^* \cdot p_T^*) \cdot (1 + r^*)^{2N} \tag{1}$$

- Another possibility is for the entrepreneur to travel on the spaceship with the goods, which would take $2\bar{n}$ years.

$$\frac{p_E^* q_T^* p_T}{p_E} \geq (c + q_T^* \cdot p_T^*) \cdot (1 + r^*)^{2N \cdot \sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

- They are two obviously contradictory criteria - which one is correct?

The answer: Opportunity costs

- To determine the correct NPV criteria, consider the opportunity costs for the Trantorian entrepreneur.
- The alternative to undertaking the interstellar transaction is to purchase a bond offering an interest rate of r^* and hold it to maturity.
- Naturally, this means that $2N$, not $2\bar{n}$, is the relevant time measure.

Theorem (The First Fundamental Theorem of Interstellar Trade)

When trade takes place between two planets in a common inertial frame, the interest costs on goods in transit should be calculated using time measured by clocks in the common frame and not by clocks in the frames of trading spacecraft

What about one-way trips?

- What if the entrepreneur considers staying on Earth, instead of returning to Trantor?
- Returning to equation (1), assuming $c = 0$ and perfectly competitive markets (zero profits):

$$\frac{p_E^*}{p_T^*} = \frac{p_E}{p_T} \cdot (1 + r^*)^{2N} \tag{3}$$

- This means prices will not equalise between both planets.
- The entrepreneur planning to emigrate to Earth could purchase the same Trantorian bond and sell it on Earth.
- The buyer of that bond can arbitrage between two options: The bond, or taking Earth goods to Trantor and selling them there.

What about one-way trips?

- As per equation (3), the value of one Trantorian good on Earth relative to its price on Trantor is $\frac{p_E}{p_E^*} \cdot (1 + r^*)^{2N}$.
- So a bond paying $\frac{p_T}{p_T^*}$ at maturity is worth $\frac{p_E}{p_E^*} \cdot (1 + r^*)^{2N}$ as well.
- The Trantonian counterpart is indifferent between both!
- The First Fundamental Theorem of Interstellar Trade holds, even with one-way trips (as long as there are complete markets).

Relaxing Interest Rate Assumption

- It is not obvious interest rates should equalise.
- For interest rates to equalise in international trade, free movement of capital suffices.
- Capital cannot move freely between planets:
  - Information (i.e. instructions) travel only at light speed.
  - Goods take time to transfer between planets (even slower than light speed).
- If interest rates are not equal, it may be that arbitrage is still not possible.

A second interstellar transaction

- A Trantorian resident considers the following project, similar to the previous one:
  1. Shipping goods to Earth.
  2. Investing the profits from the sale into a bond on Earth with maturity in K years.
  3. Buying Terran goods and shipping them back.
- For arbitrage to be impossible, this project must yield the same profits as holding a bond for the same amount of time (that is, $2N + K$ years):

$$(1+r^*)^{2N+K} = \left(\frac{p_E^*}{p_T^*}\right)\left(\frac{p_T}{p_E}\right)(1+r)^K \tag{4}$$

The Second Theorem of Interstellar Trade

- (4) must hold for arbitrage to be impossible.
- We can substitute (3) in (4) for relative prices.
- This reduces to $r = r^*$.

Theorem (The Second Fundamental Theorem of Interstellar Trade)

If sentient beings may hold assets on two planets in the same inertial frame, competition will equalize the interest rates on the two planets.

Takeaways

- **First Theorem:** even though transit time is ambiguous once ships approach c, the correct discounting horizon is always the time elapsed in the trading planets' shared frame (N), never the time elapsed aboard ship ($\bar{n}$), because the opportunity cost of capital is set by a bond sitting still back home, whose value never depended on anyone's travel time.
- **Second Theorem:** if agents on both planets can hold assets in the other's markets, ordinary no-arbitrage competition equalizes interest rates, $r = r^*$.

> "These complications make the theory of interstellar trade appear at first quite alien to our usual trade models; presumably, it seems equally human to alien trade theorists. But the basic principles of maximization and opportunity cost will be seen to give clear answers to these questions. I do not pretend to develop here a theory that is universally valid, but it may at least have some galactic relevance."