Attitudes Toward Success and Failure

Risk-attitude reversals around meaningful thresholds

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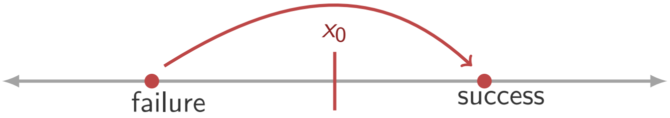
Introduction

- People often face a meaningful threshold that separates success from failure.
- Risk attitude can reverse when the expected outcome crosses that threshold.
- The paper identifies four distinct patterns of reversal and connects them to utility curvature and kinks.
- The paper shows that Prospect Theory and aspiration models can be behaviorally interpreted within expected utility.



Four situations

1. A student likely to fail takes a risky resit for even a small chance of passing.
2. A salesperson close to target takes a gamble rather than settle just below the bonus cutoff.
3. A household declines a long-shot rescue, but takes the same type of risk once success becomes likely enough.
4. A firm pursues a risky expansion only when its chance of clearing a growth target.

Primitives pt.1

- $\mathbb{R}$ is a set of real-valued outcomes.
- L is the space of lotteries over $\mathbb{R}$ with finite support, with typical element $p, q, r \in L$.
- For any $x, x' \in \mathbb{R}$, let $\Delta(x, x')$ denote the set of lotteries with support $\{x, x'\}$.
- p denote both a lottery and the probability of the high outcome x' in that lottery.
- For any $p \in L$ let p^e denote the expected value of the lottery.
- For any $x \in \mathbb{R}$, let δ_x denote the degenerate lottery that pays x with probability 1.

Decision-maker preference

- **Weak Order:** $\succsim$ is a complete and transitive preference relation over L.
- **Independence:** For any $p, q, r \in L$ and $\alpha \in (0,1)$,

$$p \succsim q \iff \alpha p + (1-\alpha) r \succsim \alpha q + (1-\alpha) r.$$

- **Archimedean Property:** For any $p, q, r \in L$ with $p \succ q \succ r$, there exist $\alpha, \beta \in (0,1)$ such that:

$$\alpha p + (1-\alpha) r \succ q \succ \beta p + (1-\beta) r.$$

- **Monotonicity:** For any $x, x' \in \mathbb{R}$ with $x' > x$ and any $p \in \Delta(x, x')$, we have

$$p \succsim \delta_x \quad \text{and} \quad \delta_{x'} \succsim p.$$

Expected Utility Representation:

There exists a continuous and strictly increasing utility function $u : \mathbb{R} \to \mathbb{R}$ such that for any $p, q \in L$,

$$p \succsim q \iff \mathbb{E}[u(X)] \geq \mathbb{E}[u(X)].$$

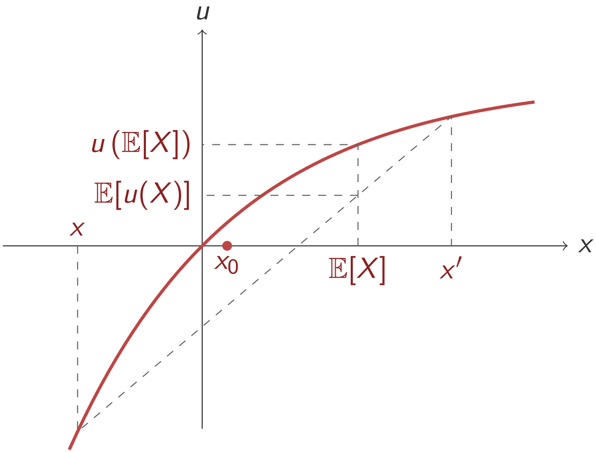
Globally concave expected utility cannot generate the reversal

$$u(x) = 1 - \mathrm{e}^{-x}$$

by Jensen's inequality,

$$u(\mathbb{E}[X]) \geq \mathbb{E}[u(X)].$$

The agent always prefers the sure expected value to the lottery.



Primitives pt.2

- The paper studies risk-attitude reversals around a meaningful exogenous threshold $x_0 \in \mathbb{R}$.
- x_0 separates success $(x' > x_0)$ from failure $(x' < x_0)$.
- Every notion will have in common the following features:
  - There is no reversal of the agent's risk attitude over lotteries whose prizes are on the same side of the reference point.
  - There is a reversal of the agent's risk attitude for lotteries across the reference point: Over such loteries, the agent would be risk averse for some and risk loving for others.
- For any $x_f, x_s \in \mathbb{R}$ where $x_0 \in (x_f, x_s)$, let:

$$\mathcal{S}_{x_f}^{x_s} := \{y \in [x_f, x_s] : \text{sign}(y - x_0) = \text{sign}(x - x_0)\}$$

- $m^- := \lim_{x \to x_0^-} u'(x)$, $m^+ := \lim_{x \to x_0^+} u'(x)$

Reversal condition

Definition 1: Same side no reversal (SSNR)

Let $x_f < x_0 < x_s$. Preferences $\succsim$ display SSNR over the interval (x_f, x_s) if:

$$\forall x \in [x_f, x_s]\,,\ \nexists x' \in \mathcal{S}_{x_f}^{x_s}(x, x_0):\ (\delta_{q^e} \succ q)\ \wedge\ (p \succ \delta_{p^e})\,,$$

for some $p, q \in \Delta(x, x')$.

Failure avoidance: “I will gamble to escape failure”

Intuition

Fix a potential failure. Even a tiny success can make the gamble attractive when failure is likely; at sufficiently high success probabilities, the agent returns to risk aversion.

Exam vignette

A student accepts a risky resit rather than a certain failing grade.

Failure avoidance formalization

Definition 2: Failure Avoidance

Preferences $\succsim$ display failure avoidance at $x_0 \in \mathbb{R}$ if $\exists x_f, x_s : x_f < x_0 < x_s$ such that:

1. $\succsim$ displays SSNR over (x_f, x_s);
2. $\forall x \in [x_f, x_0)\ \ \exists \bar{x} \in (x_0, x_s] : \forall x' \in (x_0, \bar{x}],\ \exists p, q \in \Delta(x, x')$ such that:

$$(p > q)\ \wedge\ (\delta_{p^e} \succ p)\ \wedge\ (q \succ \delta_{q^e}).$$

Theorem 1: Failure Avoidance: Representation

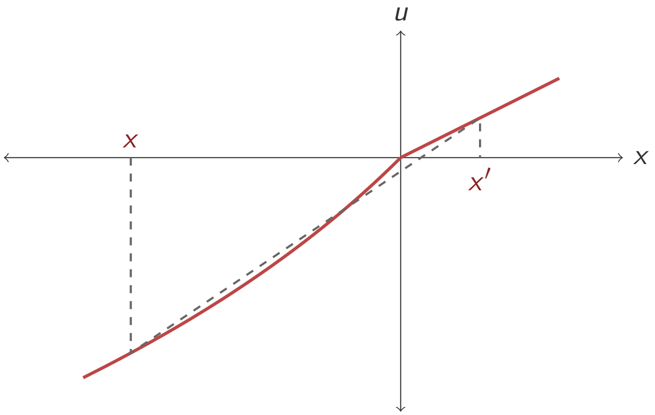
Under vNM plus monotonicity, $\succsim$ displays failure avoidance at x_0 if and only if there exists $x_f, x_s \in \mathbb{R}$: $x_f < x_0 < x_s$ such that $\succsim$ either:

1. u is continuous on (x_f, x_s), strictly convex on (x_f, x_0), either concave or convex on (x_0, x_s), and such that $m^- > m^+$.
2. u is discontinuous at x_0, it is either convex or concave on each interval (x_f, x_0) and (x_0, x_s), and such that $m^+ < \infty$.

A concrete failure-avoidance utility

$$u_{FA}(x) = \begin{cases} -\ln(1-x), & x < 0, \\ x/2, & x \geq 0. \end{cases}$$

- $u''(x) > 0$ below 0;
- $m^- = 1 > 1/2 = m^+$.



Failure avoidance: a numerical reversal

Fix $x = -1$ and $x' = 1$. Then $u_{FA}(-1) = -\ln 2$ and $u_{FA}(1) = 1/2$.

High-success lottery p

$$p = \tfrac{3}{4}\delta_1 + \tfrac{1}{4}\delta_{-1}, \qquad p^e = \tfrac{1}{2}.$$

$$\mathrm{EU}(p) = \tfrac{3}{8} - \tfrac{1}{4}\ln 2 \approx 0.202,$$

$$u_{FA}(p^e) = 0.250 > \mathrm{EU}(p),$$

$$\Rightarrow \ \delta_{p^e} \succ p.$$

Low-success lottery q

$$q = \tfrac{1}{4}\delta_1 + \tfrac{3}{4}\delta_{-1}, \qquad q^e = -\tfrac{1}{2}.$$

$$\mathrm{EU}(q) = \tfrac{1}{8} - \tfrac{3}{4}\ln 2 \approx -0.395,$$

$$u_{FA}(q^e) = -0.405 < \mathrm{EU}(q),$$

$$\Rightarrow \ q \succ \delta_{q^e}.$$

Defining reversal displayed

The probability of success falls from $p = 3/4$ to $q = 1/4$: $p > q$, risk aversion at p, and risk lovingness at q.

Success attachment: “Ending in success has special value”

Intuition

Fix a potential success. For failures sufficiently close to the threshold, the agent displays the same probability reversal: risk loving when success is unlikely, risk averse when it is likely.

Bonus vignette

A salesperson prefers a risky contract to a sure outcome just below the annual target.

Success attachment formalization

Definition 3: Success Attachment

Preferences $\succsim$ display success attachment at $x_0 \in \mathbb{R}$ if $\exists x_f, x_s : x_f < x_0 < x_s$ such that:

1. $\succsim$ displays SSNR over (x_f, x_s);
2. $\forall x' \in (x_0, x_s]\ \exists \underline{x} \in [x_f, x_0) : \forall x \in [\underline{x}, x_0)$, $\exists p, q \in \Delta(x, x')$ such that:

$$(p > q)\ \wedge\ (\delta_{p^e} \succ p)\ \wedge\ (q \succ \delta_{q^e}).$$

Theorem 2: Success Attachment: Representation

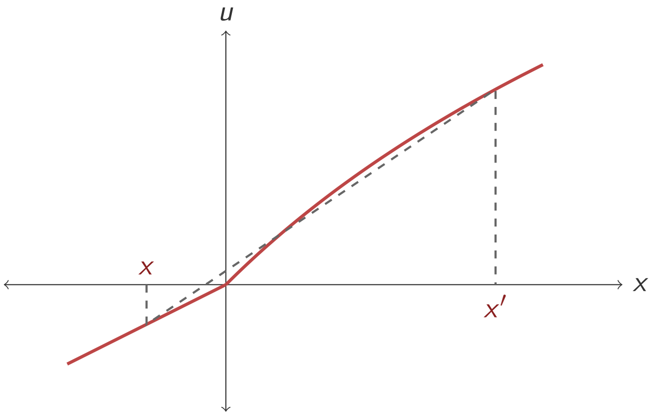
Under vNM plus monotonicity, $\succsim$ displays success attachment at x_0 if and only if there exists $x_f, x_s \in \mathbb{R}$: $x_f < x_0 < x_s$ such that either:

1. u is continuous on (x_f, x_s), strictly concave on (x_0, x_s), either concave or convex on (x_f, x_0), and such that $m^- < m^+$.
2. u is discontinuous at x_0, it is either convex or concave on each interval (x_f, x_0) and (x_0, x_s), and such that $m^+ < \infty$.

A concrete success-attachment utility

$$u_{SA}(x) = \begin{cases} x/2, & x < 0, \\ \ln(1+x), & x \geq 0. \end{cases}$$

- $u''(x) < 0$ above 0;
- $m^- = 1/2 < 1 = m^+$.



Success attachment: a numerical reversal

Fix $x = -1$ and $x' = 1$. Then $u_{SA}(-1) = -1/2$ and $u_{SA}(1) = \ln 2$.

High-success lottery p

$$p = \tfrac{3}{4}\delta_1 + \tfrac{1}{4}\delta_{-1}, \qquad p^e = \tfrac{1}{2}.$$

$$\begin{aligned} \mathrm{EU}(p) &= \tfrac{3}{4}\ln 2 - \tfrac{1}{8} \approx 0.395, \\ u_{SA}(p^e) &= 0.405 > \mathrm{EU}(p), \\ &\Rightarrow \ \delta_{p^e} \succ p. \end{aligned}$$

Low-success lottery q

$$q = \tfrac{1}{4}\delta_1 + \tfrac{3}{4}\delta_{-1}, \qquad q^e = -\tfrac{1}{2}.$$

$$\begin{aligned} \mathrm{EU}(q) &= \tfrac{1}{4}\ln 2 - \tfrac{3}{8} \approx -0.202, \\ u_{SA}(q^e) &= -0.250 < \mathrm{EU}(q), \\ &\Rightarrow \ q \succ \delta_{q^e}. \end{aligned}$$

Defining reversal displayed

Holding the success prize fixed, $p > q$, with risk aversion at p and risk lovingness at q.

Failure resignation: “I gamble only when success is likely”

Intuition

Fix a potential failure. The agent is risk averse at low success probabilities, but becomes risk loving once success is sufficiently likely.

Debt vignette

A household rejects a long-shot rescue, but accepts an investment when its chance of restoring solvency rises.

Failure resignation formalization

Definition 4: Failure Resignation

Preferences $\succsim$ display failure resignation at $x_0 \in \mathbb{R}$ if $\exists x_f, x_s : x_f < x_0 < x_s$ such that:

1. $\succsim$ displays SSNR over (x_f, x_s);
2. $\forall x \in [x_f, x_0)\ \exists \bar{x} \in (x_0, x_s] : \forall x' \in (x_0, \bar{x}], \exists p, q \in \Delta(x, x')$ such that:

$$(p < q)\ \wedge\ (\delta_{p^e} \succ p)\ \wedge\ (q \succ \delta_{q^e}).$$

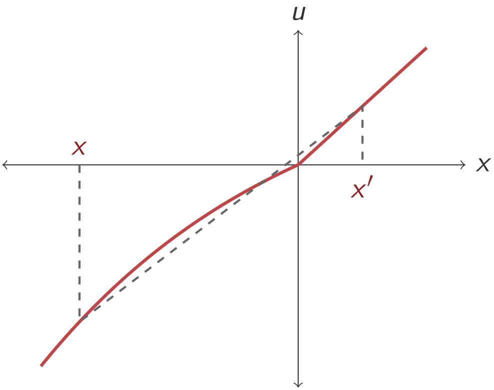
Theorem 3: Failure Resignation: Representation

Under vNM plus monotonicity, $\succsim$ displays failure resignation at x_0 if and only if there exists $x_f, x_s \in \mathbb{R} : x_f < x_0 < x_s$ such that: u is continuous on (x_f, x_s), strictly concave on (x_f, x_0), either concave or convex on (x_0, x_s), and such that $m^- < m^+$.

A concrete failure-resignation utility

$$u_{FR}(x) = \begin{cases} 1 - e^{-x}, & x < 0, \\ 2x, & x \geq 0. \end{cases}$$

- $u''(x) < 0$ below 0;
- $m^{-} = 1 < 2 = m^{+}$.



Failure resignation: a numerical reversal

Fix $x = -1$ and $x' = 1$. Then $u_{FR}(-1) = 1 - e$ and $u_{FR}(1) = 2$.

Low-success lottery p

$$p = \tfrac{1}{4}\delta_1 + \tfrac{3}{4}\delta_{-1}, \qquad p^e = -\tfrac{1}{2}.$$
$$\mathrm{EU}(p) = \tfrac{5}{4} - \tfrac{3}{4}e \approx -0.789,$$
$$u_{FR}(p^e) = -0.649 > \mathrm{EU}(p),$$
$$\Rightarrow\ \delta_{p^e} \succ p.$$

High-success lottery q

$$q = \tfrac{3}{4}\delta_1 + \tfrac{1}{4}\delta_{-1}, \qquad q^e = \tfrac{1}{2}.$$
$$\mathrm{EU}(q) = \tfrac{7}{4} - \tfrac{1}{4}e \approx 1.070,$$
$$u_{FR}(q^e) = 1.000 < \mathrm{EU}(q),$$
$$\Rightarrow\ q \succ \delta_{q^e}.$$

Defining reversal displayed

The probability of success rises from $p = 1/4$ to $q = 3/4$: $p < q$, risk aversion at p, and risk lovingness at q.

Success seeking: "I pursue the target once success is plausible"

Intuition

Fix a potential success. For failures sufficiently close to the threshold, the agent is risk averse when success is unlikely and risk loving when it becomes likely.

Growth vignette

A firm chooses risky expansion only once the probability of reaching a growth target is high enough.

Success seeking formalization

Definition 5: Success Seeking

Preferences $\succsim$ display success seeking at $x_0 \in \mathbb{R}$ if $\exists x_f, x_s : x_f < x_0 < x_s$ such that:

1. $\succsim$ displays SSNR over (x_f, x_s);
2. $\forall x' \in (x_0, x_s] \; \exists \underline{x} \in [x_f, x_0) : \forall x \in [\underline{x}, x_0)$, $\exists p, q \in \Delta(x, x')$ such that:

$$(p < q) \;\wedge\; (\delta_{p^e} \succ p) \;\wedge\; (q \succ \delta_{q^e}).$$

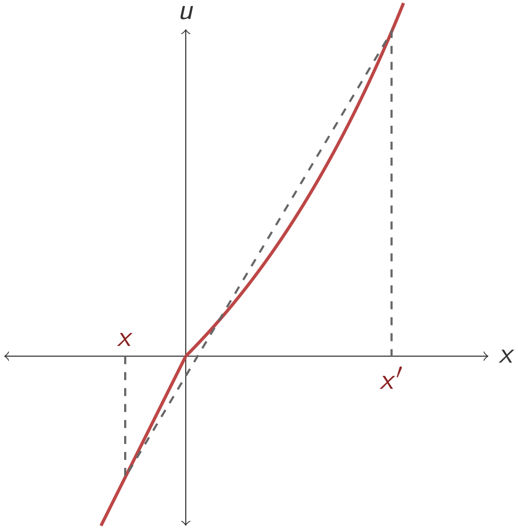
Theorem 4: Success Seeking: Representation

Under vNM plus monotonicity, $\succsim$ displays success seeking at x_0 if and only if there exists $x_f, x_s \in \mathbb{R} : x_f < x_0 < x_s$ such that: u is continuous on (x_f, x_s), strictly convex on (x_0, x_s), either concave or convex on (x_f, x_0), and such that $m^- > m^+$.

A concrete success-seeking utility

$$u_{SS}(x) = \begin{cases} 2x, & x < 0, \\ e^x - 1, & x \geq 0. \end{cases}$$

- $u''(x) > 0$ above 0;
- $m^- = 2 > 1 = m^+$.



Success seeking: a numerical reversal

Fix $x = -1$ and $x' = 1$. Then $u_{SS}(-1) = -2$ and $u_{SS}(1) = e - 1$.

Low-success lottery p

$$p = \tfrac{1}{4}\delta_1 + \tfrac{3}{4}\delta_{-1}, \qquad p^e = -\tfrac{1}{2}.$$

$$\mathrm{EU}(p) = \tfrac{1}{4}e - \tfrac{7}{4} \approx -1.070,$$

$$u_{SS}(p^e) = -1.000 > \mathrm{EU}(p),$$

$$\Rightarrow \ \delta_{p^e} \succ p.$$

High-success lottery q

$$q = \tfrac{3}{4}\delta_1 + \tfrac{1}{4}\delta_{-1}, \qquad q^e = \tfrac{1}{2}.$$

$$\mathrm{EU}(q) = \tfrac{3}{4}e - \tfrac{5}{4} \approx 0.789,$$

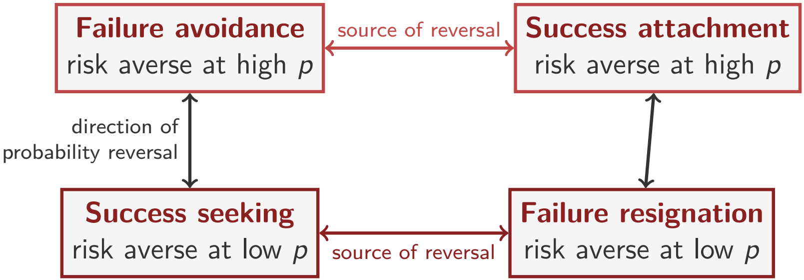
$$u_{SS}(q^e) = 0.649 < \mathrm{EU}(q),$$

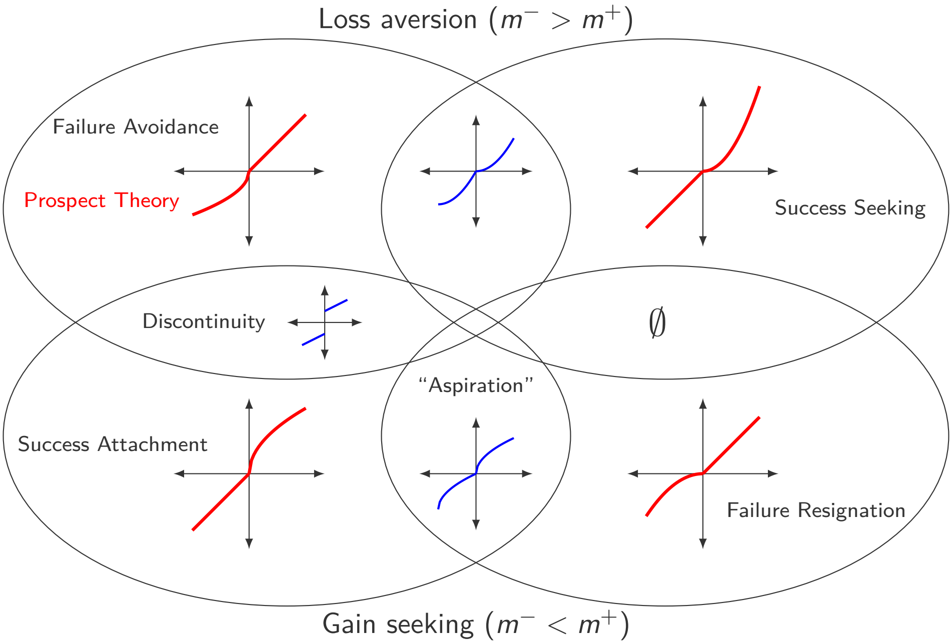
$$\Rightarrow \ q \succ \delta_{q^e}.$$

Defining reversal displayed

Holding the success prize fixed, $p < q$, with risk aversion at p and risk lovingness at q.

The four attitudes form a map, not a list





Loss aversion ($m^- > m^+$)
Failure Avoidance
Prospect Theory
Success Seeking
Discontinuity
$\emptyset$
"Aspiration"
Success Attachment
Failure Resignation
Gain seeking ($m^- < m^+$)

The map yields sharp compatibility results

Failure avoidance + success attachment

With continuous utility, they require opposite kink directions. They can coexist only through a discontinuity at x_0.

Failure resignation + success seeking

They are mutually exclusive: their required reversal patterns cannot be jointly generated.

Interpretation

Observing a pattern of cross-threshold risk taking rules out some psychological mechanisms.

What the paper adds

1. A common language for four distinct threshold-driven reversals in risk attitude.
2. Utility representations that connect those behaviors to curvature and kinks.
3. A behavioral interpretation of Prospect Theory and aspiration models within expected utility.
4. A warning: loss aversion, failure avoidance, and risk aversion are not interchangeable concepts.

Thresholds do not merely label outcomes; they can reorganize risk taking.